Horwood Critique Article

Capacity Expansion Modeling of Water Supply in a Planning Support System for Urban Growth Management

Hyong-Bok Kim and Lewis D. Hopkins

Abstract: *A planning support system enhances our ability to use infrastructure capacity expansion as an urban growth management strategy. This paper reports the development of a capacity expansion model as part of the continuing development of such a planning support system (PEGASUS: Planning Environment for Generation and Analysis of Spatial Urban Systems) to incorporate water supply. This system is designed from the understanding that land use and development drive the demand for infrastructure and infrastructure can have a significant influence on the ways in which land is developed and used. A water-distribution network analysis model and a network capacity expansion model can address the dynamic interdependence between water planning and land use planning. While the water-distribution network analysis model evaluates the performance of generated networks, the capacity expansion model chooses alternatives to meet expanding water needs. GIS provides a tool for estimating the volume of demanded water and showing results of the capacity expansion model.*

A planning support system (PSS) (Harris and Batty 1993) combines spatial analysis functions, GIS, and user interface and modeling functions as developed in the decision support system (DSS) literature (e.g., Davis-Stemp *et al.* 1986). We focus on capacity expansion of water supply rather than the usual focus on transportation. A capacity expansion model for water planning seeks to find the capacity of water-distribution networks at each stage (year) to meet changing water demand and to manage urban growth. This paper reports our progress toward an operational PSS that addresses various infrastructure investment questions related to urban growth.

Hyong-Bok Kim is acting director of Land Research Institute at Korea Land Development Corporation in Seoul, Korea. He received a Ph.D. in the Department of Urban and Regional Planning at the University of Illinois at Urbana-Champaign. He currently focuses on capacity expansion for water and wastewater planning in a planning support system.

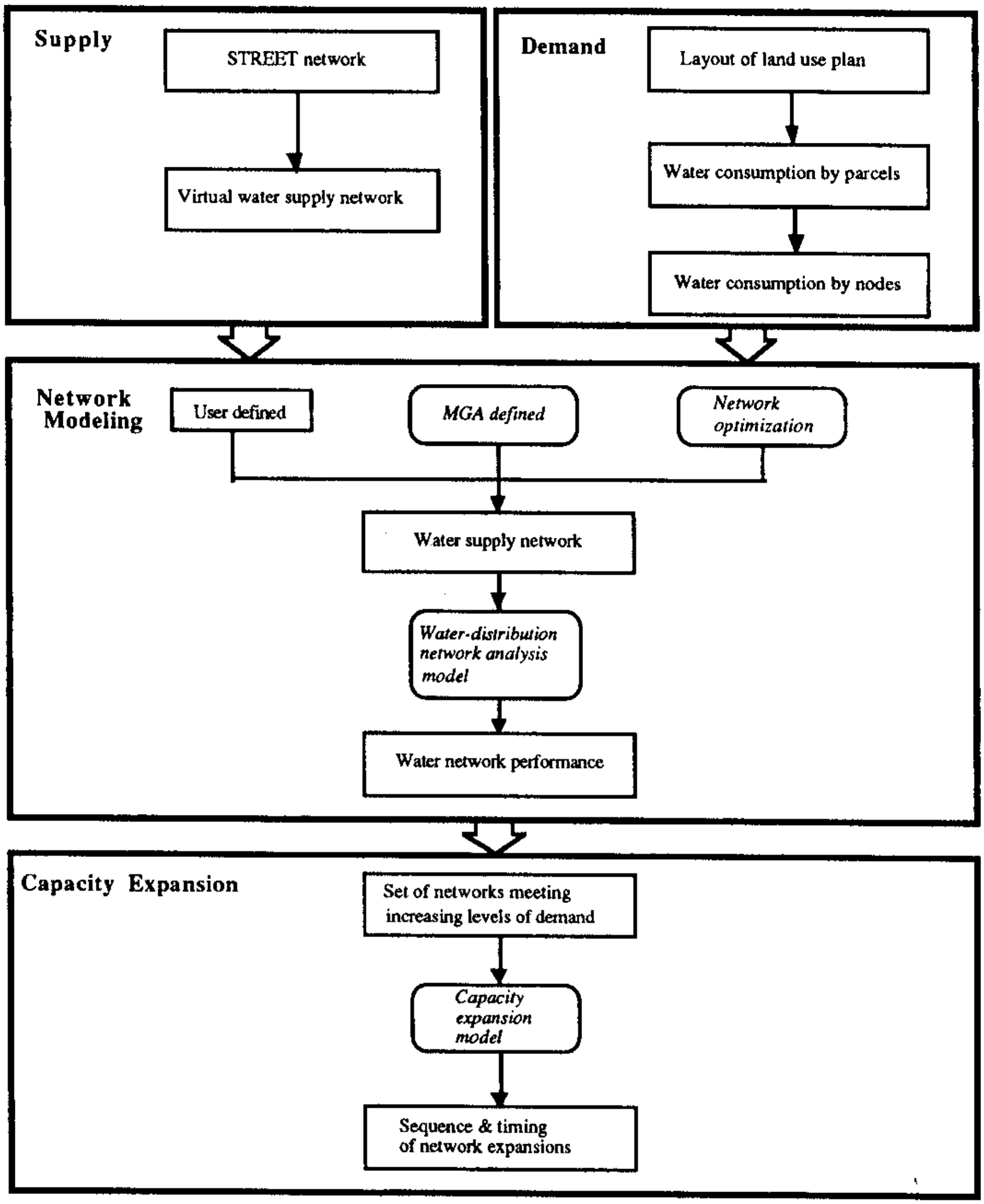
Lewis D. Hopkins is professor and head of the Department of Urban and Regional Planning at the University of Illinois at Urbana-Champaign. His research interests include design and development of planning support systems and investigation into the logic of planning for urban development.

Planning Support System for Capacity Expansion

The PSS for capacity expansion modeling of water supply is composed of four major parts (Figure 1). The first is *Demand.* In this step, we calculate the amount of water demanded per parcel and aggregate this demand to nodes on a water-distribution network using GIS. The second is *Supply.* In this step, a virtual water-distribution network, which includes all possible links, is generated based on street network and surface water data. The third is *Network Modeling.* Water-distribution network alternatives for given levels of water demand are identified by the user, by using modeling-to-generate-alternatives (MGA) (Hopkins, Brill and Wong 1982), or by a network optimization model. A network analysis model checks water pressures more completely than is possible within the network optimization model. The fourth is *Capacity Expansion.* A capacity expansion model generates capacity expansion alternatives that sequence and set the timing of network projects to meet changing demand. We present here a water-distribution network optimization model, a network analysis model, and a capacity expansion model. These models are linked to a GIS, which in this study was ARC/INFO (ESRI 1992).

We consider price effects, water pressure, and peaking of demand. Water consumption will increase as the

FIGURE 1. Implementaion Procedure of PSS for Capacity Expansion Modeling of Water Supply.



price per unit decreases (Kindler and Russel (eds.) 1984, pp.7–9) and as water pressure increases (McGhee 1991, pp.11–12). Hanke and Boland (1971) and Kindler and Russel (1984) summarized literature addressing the price-demand relationship for municipally supplied water, all of which concluded that the quantity of water used is somewhat sensitive to the price charged. The amount of water consumed also depends on available residual water pressure (Maddaus 1987) in the network because higher pressure delivers more water per unit time and some uses are sensitive to duration of flow. In addition, two fluctuation factors are used to derive peak water consumption from annual average daily consumption. The size of a community determines the ratio of maximum daily flow to annual average daily flow. The ratio of maximum peak flow to maximum daily flow determines network design capacity (McGhee 1991, p.13).

Water Network Models

Water-Distribution Network Optimization Model

The water-distribution network optimization model following (Brooke, Drud and Meeraus 1985) chooses link sizes using discontinuous nonlinear programming. The GAMS program (General Algebraic Modeling System) developed by Brooke, Kendrick and Meeraus (1992) can be used to solve such a discontinuous nonlinear programming problem.

$$\text{Minimize } Z = cost\sum_{ij} link_{ij} \cdot length_{ij} \cdot SIZE_{ij}^{cpow} \qquad (1)$$

Subject to

$$\sum_{j}\left(QTY_{ji} \cdot link_{ji} - QTY_{ij} \cdot link_{ij}\right) + sup_i = con_i \quad \forall i \; (2)$$

$$HEAD_i - HEAD_j = hloss \cdot length_{ij} \cdot abs\left(QTY_{ij}\right)^{qpow-1} \cdot QTY_{ij} / SIZE_{ij}^{dpow} \qquad \forall_i \forall_j \qquad (3)$$

$$SIZE_{ij} \rangle 0 \qquad (4)$$

where
- i, j from-node and to-node of link,
- $cost$ cost per unit of length and diameter,
- $SIZE_{ij}$ link size (pipe diameter) with from-node i and to-node j,
- $cpow$ power on diameter for cost, an economies of scale parameter,
- QTY_{ij} the amount of water flowing from-node i to-node j,
- $link_{ij}$ 0-1 variable defining existence of flow in link in direction from node i to node j,
- sup_i the amount of water supplied at node i,
- con_j the amount of water consumed at node j,
- $length_{ij}$ the length of link from node i and to node j,
- $HEAD_i$ water head of node i,
- $HEAD_j$ water head of node j,
- $hloss$ constant in pressure loss equation,
- $qpow$ parameter on flow in pressure loss equation.
- $dpow$ parameter on size in pressure loss equation.

In this model the objective function, Equation 1, is to minimize construction costs of the network. A 0-1 variable, $link_{ij}$ means that the decision variable $SIZE_{ij}$ is included in the summation if the $link_{ij}$ exists. Construction costs are a function of link size and length (Orth 1986, pp. 71–75; Singh and Adams 1980, p.44). The first constraint, Equation 2, is a continuity equation that ensures that at each node in the network the sum of inflow equals the sum of outflow. The second constraint, Equation 3, is a water pressure loss equation that ensures that the head losses through different paths between two nodes of a network are equal. Equation 4 is a non-negativity constraint that ensures that the size of every link must be positive.

Nonlinear problems can be solved more easily if guesses are supplied for the values of some variables. There are two reasons to set bounds for variables. The first is to "prevent undefined operations." The second is to "ensure that variables stay in a region that makes sense." (Brooke, Kendrick and Meeraus 1992, pp.156–157) This nonlinear model is thus solved using lower and upper bounds and initial values of link size and water pressure. The results from the model are converted from continuous pipe diameters into available integer sizes. The integer pipe size is used in recalculating construction costs. The integer pipe size is also provided to the water-distribution network analysis model for the purpose of calculating water velocity per link and water pressure per node more accurately than is possible in the network optimization model.

Water-Distribution Network Analysis Model

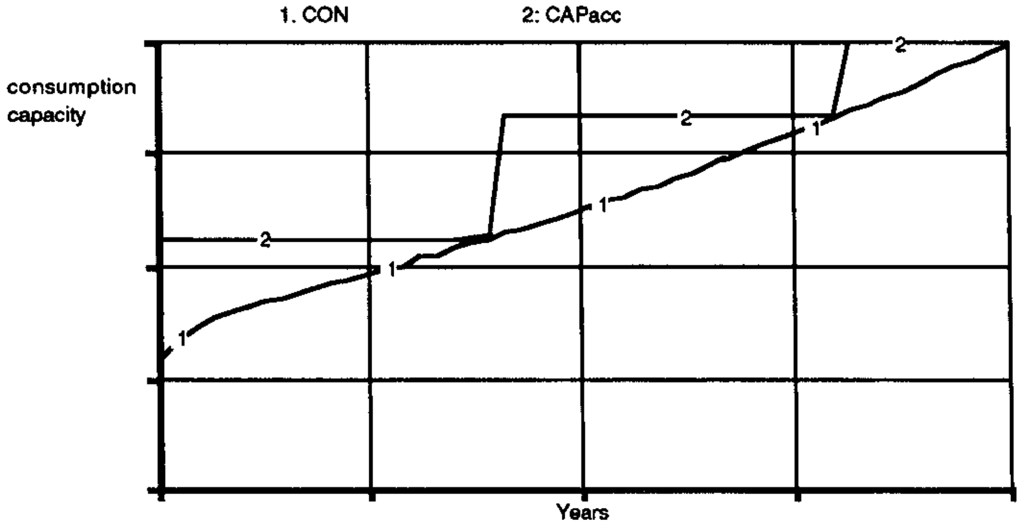
The water-distribution network analysis model simulates flow and water pressure for a single point in time, in order to determine the water pressures at nodes and flows in links given a distribution network and pipe sizes assigned from the network optimization model. For a detailed description, see Cesario (1991). Among many methods to calculate water pressure, the Hardy-Cross method (1936) was chosen because of its widespread acceptance and simplicity (Morgan and Goulter 1985). The pipe sizes can be adjusted as necessary to ensure that the pressures at the various nodes and the velocities in the various pipes meet the established velocity criteria. Velocity normally does not exceed 1 m/sec (McGhee 1991).

Capacity Expansion Modeling

The capacity expansion model provides information about how to expand the water-distribution networks so that they meet increasing demand over time. This involves deciding the expansion size (sizing), expansion times (timing), and expansion capacity types (Luss 1982). The design period of water networks (i.e., the length of time from capacity construction to full use of capacity) depends on discount rate, projected demand, construction and management costs, and the ease of capacity expansion. We make the following assumptions pertinent to the installation of water capacity over time in order to simplify the dynamic capacity expansion model:

1. Capacity is durable; that is, capacity once installed has an infinite life.

FIGURE 2. A Staged Capacity Expansion Policy.



2. Consumption cannot exceed capacity.
3. Negative demand increments are not allowed.
4. Demand is deterministically forecasted.
5. There are no holding costs for excess capacity.
6. The addition of capacity to existing capacity is additive.
7. There are no lag times from decision to service provision; that is, no construction time.

We will relax some of these assumptions in further development of the system.

A continuous increase in consumption (1: CON in Figure 2) is derived from an exogenous population growth function as explained below. The lumpy investments in discrete capacity expansion are shown as 2: CAPacc in Figure 2. The capacity expansion model seeks least-cost scheduling (Knudsen and Rosbjerg 1977) of network projects in order to meet demand growth considering discounted rate of return of capital and operation and management costs. A least cost schedule of water-distribution network projects includes optimal selection and sequencing of potential projects. In this study, the size and sequence of water projects are chosen based on simulation results for the state equations. The present capacity expansion model thus determines only the timing of water projects.

The procedure used to formulate a dynamic capacity expansion model derives from optimal control theory (Intriligator 1971, pp.344–369; Wymer 1994) and capacity expansion (Luss 1982; Freidenhelds 1981; Manne 1961). An objective functional, 2 + n equations of motion, and one capacity constraint can be formulated as follows (Figure 3). The circle number in Figure 3 is the equation number to be explained below.

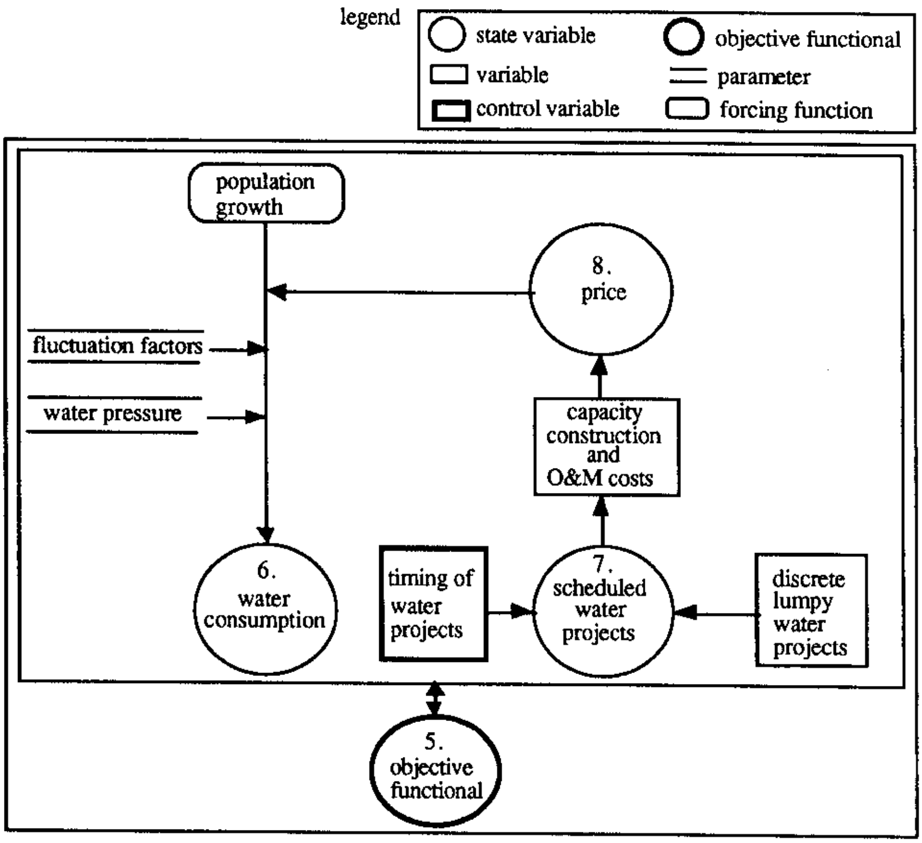
Specification of the Objective Functional

The objective functional reflects the preferences of a decision-maker and measures the degree of effectiveness of each control action. A planning horizon of 50 years is used because this will include a reasonable number of projects and the present worth of projects becomes very small in 50 years. The planning horizon is the time period of effects considered in devising the plan and the period of the plan is the period during which the plan is used to guide decisions (Intriligator 1986; 1989).

In this study we begin by assuming that a water planner seeks to minimize the costs of providing demanded water. Costs include construction and operation and maintenance (O&M) costs. At present, the capacity expansion model assumes a given sequence of given network projects. Simulation of state equations by STELLA (Peterson and Richmond 1994) provides us with an approximate trend of water consumption. Based on the simulated water consumption, a sequence of water-distribution network projects can be chosen heuristically by the PSS user. For example, short-term projects are preferred under high discount rates, while long-term projects are preferred under low discount rates. Using such heuristic knowledge, the user generates a sequence of water-distribution projects. The construction costs ($CONST_i$) of projects are computed as shown in the network optimization model described above. O&M costs are contingent on construction costs (Orth 1986, p. 71).

$$\text{Minimize } J = \int_{t_0}^{t_l} \left[e^{-dt} \cdot COST\ (t)\right]^2 dt \qquad (5)$$

FIGURE 3. Dynamic Capacity Expansion Modeling



where

$$COST(t) = \sum_{i=1}^{n}(1+\alpha_1)\cdot CONST_i \cdot swtch\left(u_i^* - \epsilon\right)$$

$COST(t)$	construction and O&M costs of water-distribution network projects commissioned as of time t (million\$),
$CONST_i$	construction costs of i^{th} project (million\$),
$swtch\{f(x)\}$	a switch function such that $S\{f(x)\} = 1$ if $f(x) \geq 0$ and 0 otherwise,
u_i^*	a control variable determining the timing of the i^{th} project,
ε	a small number to ensure that $CONST_i$ is not set until the i project is commissioned,
t_0 and t_t	beginning and end of the planning horizon,
d	discount rate,
α_1	operation and management cost relative to construction costs,

Specification of the State Equations (Equations of Motion)

The state equations include 2 + n first order differential equations: one water consumption, one price, and n scheduled lumpy water projects equations.

1) Water Consumption

Population growth rate, initial population, and water consumption per person per day yield annual average daily water consumption. The initial population is calculated by GIS from land use data. The distribution of the population increase will be determined by GIS in future work. Population increases exponentially and uniformly over space in the current model.

Peak water consumption is calculated by multiplying annual average daily water consumption by daily maximum and daily peak fluctuation factors. In addition, water consumption is affected by price elasticity and the ratio of average water pressure and allowable minimum water pressure. The water pressure at nodes for particular water projects is determined by the water-distribution network analysis model as described above. Equation 6 states that the change in the rate of water consumption, $dCON(t)$, depends on the difference between the desired rate of water consumption and the rate of water consumption with a time lag coefficient of adjustment.

$$dCON(t)/dt = \lambda_1 \cdot (\hat{CON}(t) - CON(t)) \qquad (6)$$

where $CON(t) = factor_{atd} \cdot factor_{dth} \cdot tpcd \cdot pop(t) \cdot (pratio(t))^{\beta_1} \cdot (1 + (Head_{avg}(t) - Head_{ami}) \cdot change/diff)$,

$pop(t) = pop_{orig} \cdot \exp^{rate \cdot t}$,

- $\hat{CON}(t)$ the amount of desired water consumption at time t (1,000 MTD),
- $CON(t)$ the amount of water consumption at time t (1,000 MTD),
- λ_1 an adjustment coefficient for water consumption,
- $factor_{atd}$ the ratio of maximum daily flow to annual average daily flow,
- $factor_{dth}$ the ratio of daily peak flow to maximum daily flow,
- $pop(t)$ population as of time t,
- $poporig$ population at time t_0 in study area,
- $rate$ population growth rate per year,
- $tpcd$ unit water consumption per person per day (1,000 MTD),
- $pratio(t)$ the ratio of water price to a cost of living index,
- $Head_{avg}(t)$ average residual water pressure in the study area at time t (kg/cm^2), which is determined by the water-distribution network analysis model,
- $Head_{ami}$ allowable minimum residual water pressure (kg/cm^2), an engineering design standard,
- β_1 price elasticity coefficient of water demand,
- $change$ % change in annual average daily water use for maximum allowable pressure (from empirical studies- e.g. Maddaus, 1987),
- $diff$ maximum pressure difference (kg/cm^2).

2) Scheduled Lumpy Water-Distribution Network Projects

In the case of water-distribution networks, capacity is added in lumps, that is discrete projects, because of the economies of scale in building facilities. In order to simplify solution of the problem, the size and sequence of projects are at present taken as given from an interactive exploration of possible projects carried out using the modeling tools for network project design described above.

Let an accumulated capacity of water projects be $CAP_{acc}(t)$ and let a sequence of n predefined, discrete water network projects be $CAP_1, CAP_2, \ldots CAP_n$. Let all the $z_i(t) = 0$ to initialize all values of $z_i(t)$. Every predefined project has one first-order differential equation. The first-order differential equation deals with the timing of the n water projects.

$$dz_1/dt = swtch\{u_1^* - \varepsilon\} \qquad (7a)$$

$$dz_2/dt = swtch\{z_1^* - \varepsilon\} \cdot swtch\{u_2^* - \varepsilon\} \qquad (7b)$$

$$dz_3/dt = swtch\{z_2^* - \varepsilon\} \cdot swtch\{u_3^* - \varepsilon\} \qquad (7c)$$

$$dz_n/dt = swtch\{z_{n-1}^* - \varepsilon\} \cdot swtch\{u_n^* - \varepsilon\} \qquad (7d)$$

The zero-order differential equation that defines cumulative capacity of water projects at time t is;

$$CAP_{acc}(t) = \sum_{i=1}^{n} swtch(z_i - \epsilon) \cdot CAP_i \qquad (7e)$$

where

- $z_i(t)$ a set of indicator variables that keep the information about whether the trigger function, $f(x)$, has been set in the past,
- ε a small number to ensure that z_i is not set until CAP_i is commissioned,
- $CAP_{acc}(t)$ cumulated capacity of projects at time t (1,000 MTD),
- CAP_i capacity of i^{th} water project.

3) Price Per Unit Water Supply

Our eventual intent is to make unit water price a control variable so as to use price as a means to help balance continuous increases in demand with lumpy increases in supply. In the simulation model presented below, price is endogenously determined as a function of the cost of supply. Price per day for water is set just sufficient to recover capital and operating costs for the current level of capacity. The number of years for capital recovery might be set to the number of years over which construction bonds are amortized.

$$dPRICE(t)/dt = \lambda_2 \cdot (\hat{PRICE}(t) - PRICE(t)) \qquad (8)$$

where

$$\hat{PRICE}(t) = CRF \cdot \sum_{t_0}^{t} COST(t)/(CON(t) \cdot 365)$$

TABLE 1. Initial Values of State and Exogenous Variables and Parameters for Simulation

Variable Name	Value	Variable Name	Value
CON(0)	49,560*MTD*	PRICE(0)	0.8$/*MTD*
λ_1	1.0	λ_2	1.0
rate	1.5%/*year*	*tpcd*	0.004 *MTD*
pop_{origin}	50,795	*d*	5%/*year*
$factor_{atd}$	1.5	$factor_{dth}$	1.5
α_1	0.2	$Head_{ami}$	1.5 kg/cm^2
change	6%	*diff*	2.31 kg/cm^2

$\hat{PRICE}(t)$ the desired unit water price at time t ($1,000/MTD),
$PRICE(t)$ the unit water price at time ($1,000/MTD),
λ_2 an adjustment coefficient for water price,
CRF the capital recovery factor—annual payment to repay single lump-sum payment, $CRF = d(1+d)^p/(1+d)^p - 1$,
p amortization period of construction bonds.

4) Capacity Constraint

Accumulated capacity of water projects commissioned by time t must be at least equal to water consumption at time t.

$$CAP_{acc}(t) - CON(t) \geq 0 \quad \forall t \qquad (9)$$

Implementation Example

The PSS is intended to support exploration of alternatives using the set of models to consider and create alternatives. None of the models is sufficient by itself to address the entire water supply network capacity expansion problem.

Simulation of State Equations

The trend of water consumption can be taken as an estimate of water demand for the water-distribution network analysis model. For this implementation example, the value of the price elasticity coefficient β_1 is –0.231 (Hanke, 1978). For a given network, the water-distribution network analysis model calculates average water pressure at nodes in the study area. Average water pressure at a given time is dependent on water consumption at nodes and on network link size. Water consumption is obtained by simulation of the state equations and network size is obtained based on the capacity of three predefined water-distribution network projects by the water-distribution network optimization model. Table 1 shows initial values of state and exogenous variables, and parameters.

In this illustration, water consumption increases from 49,560 to 100,000 MTD. Given this growth of demand,

FIGURE 4. A Staged Capacity Expansion Policy for Simulation.

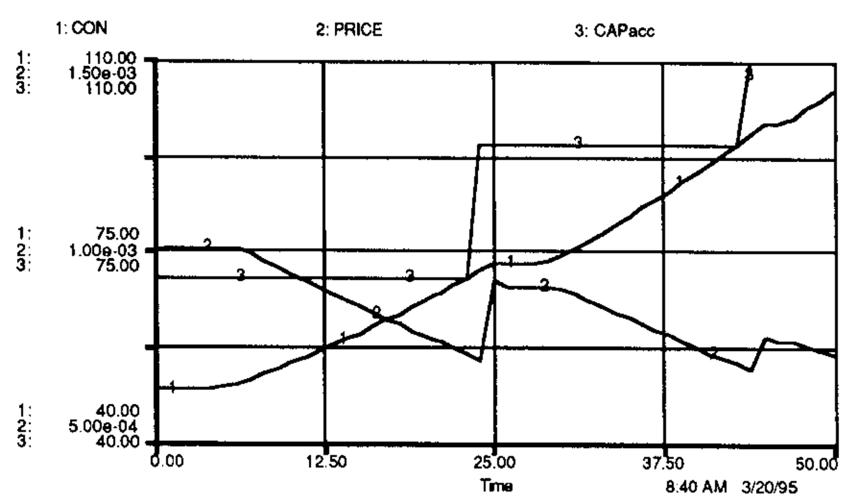


TABLE 2. Timing Combinations for Sensitivity Analysis.

Capacity	Timing	Option 1	Option 2	Option 3	Option 4
70,000 MTD	t_1	0	0	0	0
20,000 MTD	t_2	20	15	10	5
15,000 MTD	t_3	40	30	20	10

three projects, CAP_1, CAP_2, and CAP_3 were defined with 70,000; 25,000; and 15,000 MTD respectively. Figure 4 shows how timing of these second and third projects was determined. The capacity of the first water project of 70,000 MTD yields sufficient water consumption (1:CON) until year 24. At year 24, no excess capacity is available. Then the second water project of capacity 25,000 MTD is added to the cumulative capacity (3: CAPacc). Likewise, at year 44, the third project of 15,000 MTD is commissioned to meet water consumption. Price (2:PRICE) declines generally because of economies of scale in water projects, except when new water projects are constructed.

Sensitivity Analysis of State Equations

Sensitivity analysis of state equations can show the robustness and generalizability of the model by means of comparative dynamics. Comparative dynamics examine the changes in solutions of a model with respect to a change of variables. The variable selected for the sensitivity analysis is timing of water projects. Table 2 shows four timing combinations for sensitivity analysis. In Table 2, t_1, t_2 and t_3 are the timing of first (CAP_1), second (CAP_2) and third (CAP_3) projects. The timing (t_1) of initial project for every option is 0. The timing (t_3) of the third project for option 1 to option 4 is consistently twice that (t_2) for the second project.

Table 3 shows the results of sensitivity analysis with respect to the change of timing. As the time between projects decreases, the net present worth of construction and O&M costs increase, which increases water price, which decreases water consumption.

Conclusions

The PSS for capacity expansion modeling of water supply provides the user with several tools:

1. aggregating water consumption by nodes and generating a virtual water network using GIS;
2. identifying water-distribution network alternatives by user definition, the water network optimization model, or MGA;
3. evaluating the performance of water networks using the network analysis model;
4. choosing the capacity expansion policy to meet water demand using the capacity expansion model; and
5. showing results of capacity expansion model using GIS.

Only timing of water-distribution network projects is considered in this paper. We are extending the capacity expansion model to determine the size of projects and schedule projects, and to use water price as a policy variable.

TABLE 3. Sensitivity Analysis Results.

	Option 1	Option 2	Option 3	Option 4
$e^{-dt} . COST(t)$	160.78 mil$	174.26 mil$	186.84 mil$	204.84 mil$
CON(10)	54,230 MTD	54,230MTD	54,230MTD	51,260MTD
PRICE(10)	0.9389 $/MTD	0.9389$/MTD	0.9389$/MTD	1.2804$/MTD
CON(20)	65,890 MTD	61,310 MTD	61,910 MTD	58,340 MTD
PRICE(20)	0.7730 $/MTD	1.0769$/MTD	1.1019$/MTD	1.3093$/MTD
CON(30)	73,800 MTD	73,780MTD	70,910MTD	70,900MTD
PRICE(30)	0.9067 $/MTD	0.9071$/MTD	1.0775$/MTD	1.0776$/MTD
CON(40)	89,670 MTD	86,180MTD	86,170MTD	88,660MTD
PRICE(40)	0.7464 $/MTD	0.8865$/MTD	0.8866$/MTD	0.8866$/MTD
CON(50)	104,740 MTD	104,730MTD	104,730MTD	104,730MTD
PRICE(50)	0.7294 $/MTD	0.7295$/MTD	0.7295$/MTD	0.7295$/MTD

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